Student Number ....

Barker College

Mathematics

2007
TRIAL
HIGHER SCHOOL
CERTIFICATE

Staff Involved:

- JML* • LMD
- AJD* • MRB
- RMH • JM
- GDH • EAS
- PJR • JGD

AM TUESDAY 31 JULY

155 copies

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Write your Barker Student Number on all pages of your answers
- Board-approved calculators may be used
- Start each question on a NEW page
- A table of standard integrals is provided at the back of this paper which may be detached for your use
- All necessary working must be shown in every question
- Marks may be deducted for careless or badly arranged working
- Write on one side of the page only

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value

Total marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question on a new page.

Marks

Question 1 (12 marks)

(a) Evaluate $\sqrt{\dfrac{3\cdot8^3 \times 93}{3518}}$ correct to two decimal places. **2**

(b) Make 'a' the subject of the formula $s = ut + \dfrac{1}{2}at^2$ **2**

(c) Show on a number line the solution of $|2m - 1| \leq 5$ **2**

(d) If $G(x) = 1 - x^2$, evaluate $G(3) - G(0)$ **2**

(e) Factorise completely $3x^2 - 12y^2$ **2**

(f) The probability of gaining relief from a particular illness with antibiotic A is $\dfrac{7}{10}$, **2**

whilst with antibiotic B, the probability of gaining relief is $\dfrac{3}{4}$.

One cold sufferer takes A, whilst another takes B.

What is the probability that only one cold sufferer gains relief?

Question 2 (12 marks) **[START A NEW PAGE]** **Marks**

(a) Find the equation of the tangent to the curve $y = x^2 - 3x + 2$ at the point (3, 2) **2**

(b) Differentiate with respect to x:

(i) $\tan(\log_e x)$ **2**

(ii) $x\sqrt{(1 + x)}$ **2**

(c) (i) Find a primitive function of $\dfrac{2x}{x^2 + 1}$ **1**

(ii) Evaluate $\displaystyle\int_{\frac{\pi}{3}}^{\pi} \cos\left(\frac{1}{2}x\right) dx$ **2**

(d) Given that $f(x) = x^2 + x$, find the values of b for which $f''(b) = f(b)$ **3**

Question 3 (12 marks) **[START A NEW PAGE]** **Marks**

(a) Evaluate $\log_2 48 - \log_2 4 - \log_2 3$ **2**

(b) Two perpendicular lines $3x + 2y = 12$ and $2x + ay = b$ intersect at the point (2, 3). Find the values of a and b. **2**

(c) The sum of the first three terms of an arithmetic sequence is 27. **3**
The sum of the next three terms is 63.
Find the first term and common difference.

(d) Evaluate $\displaystyle\sum_{r=1}^{20}(2r + 1)$ **2**

(e) By first writing the recurring decimal $0 \cdot 303030....$ as an infinite geometric series, use your knowledge of geometric series to express it as a rational number in its simplest form. **3**

Question 4 (12 marks) **[START A NEW PAGE]** **Marks**

The coordinates of the points A, B and C are (0, 2), (4, 0) and (6, –4) respectively.

(i) Find the length AB. **2**

(ii) Find the gradient of AB. **1**

(iii) Show that the equation of the line ℓ, drawn through C and parallel to AB is

$x + 2y + 2 = 0$ **2**

(iv) Find the coordinates of D, the point where ℓ intersects the x-axis. **2**

(v) Find the perpendicular distance of the point A from the line ℓ. **2**

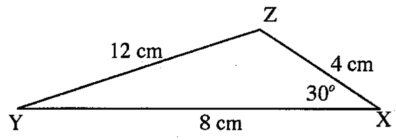
(vi) Hence, or otherwise, find the area of the quadrilateral ABCD. **3**

Question 5 (12 marks) **[START A NEW PAGE]** **Marks**

(a) Show that $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$ **2**

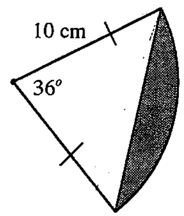
(b) Solve $2\cos x = \sqrt{3}$, where $0 \leq x \leq 2\pi$ **2**

(c)



Find the area of the triangle XYZ. **2**

(d)

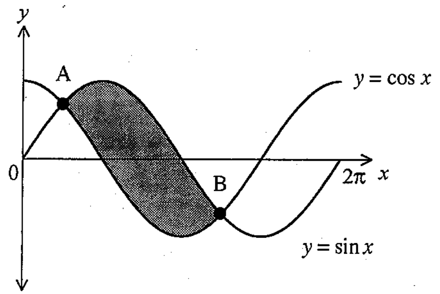


The diagram shows the area of a minor segment of a circle with radius $10cm$. **2**
Find the area of this minor segment.

Question 5 continues on next page

Question 5 (continued) **Marks**

(e)



The diagram shows the graphs of the curves $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq 2\pi$

(i) Show that the x-coordinates of the two marked points A and B are $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ respectively. **1**

(ii) Calculate the shaded area leaving your answer in surd form. **3**

End of Question 5

Marks

Question 6 (12 marks) **[START A NEW PAGE]**

(a) Find all values of k for which the quadratic equation $kx^2 - 8x + k = 0$ has real roots. **3**

(b) A function $f(x)$ is defined by $f(x) = 2x^3 + ax^2 + bx + 3$

(i) If this function $y = f(x)$ has stationary points when $x = 1$ and $x = -2$, show that $a = 3$ and $b = -12$. **2**

(ii) Hence, find the coordinates of the stationary points of $y = f(x)$ and determine their nature. **3**

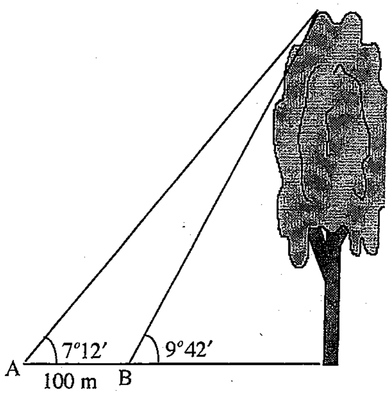
(iii) Hence, find the coordinates of the point of inflexion. **2**

(iv) Hence, sketch the graph $y = f(x)$ identifying all key features, i.e showing the turning points and the point of inflexion. **1**

(v) For what values of x is $f(x) = x^3 + 3x^2 - 12x + 3$ concave up? **1**

Question 7 (12 marks) **[START A NEW PAGE]** **Marks**

(a)



The diagram above was sketched by a surveyor, who measured the angle of elevation of the top of a tree on the other side of a river to be $7°12'$ from point A.

From point B, 100 metres directly towards the tree from A, the angle of elevation of the top of the same tree was $9°42'$.

(i) Show that the height of the tree can be expressed in the form **3**

$$h = \frac{100 \sin 7°12' \sin 9°42'}{\sin 2°30'}$$

(ii) Calculate the height of the tree correct to three significant figures. **1**

(b) (i) Find the x-intercepts of the curve $y = 2x - x^2$ **1**

(ii) The area bounded by the curve $y = 2x - x^2$ and the x-axis is rotated about the x-axis. **3**

Find the volume of the solid of revolution formed, leaving your answer in terms of π.

Question 7 continues on next page

(c) Consider the function given by $f(x) = \sin^2 x$.

(i) Copy and complete the following table onto **your answer page**. **2**

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$y = f(x)$					

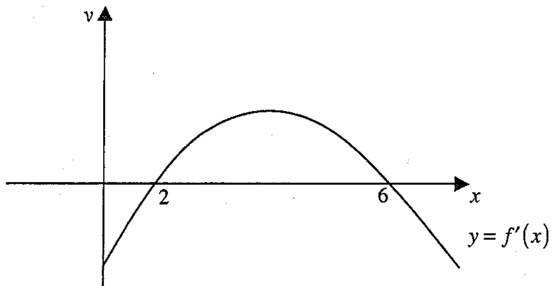
(ii) Apply Simpson's rule with five function values to find an approximation to $\int_0^{\pi} \sin^2 x \, dx$, leaving answer in exact form. **2**

End of Question 7

Marks

Question 8 (12 marks) **[START A NEW PAGE]**

(a)

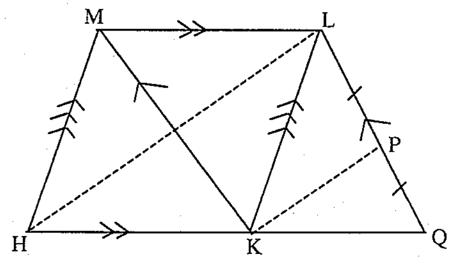


The diagram shows the graph of the gradient function of the curve $y = f(x)$.

(i) For what values of x does $y = f(x)$ have a local minimum? Justify your answer. **2**

(ii) Draw a possible sketch of the curve $y = f(x)$. **1**

(b)



HKLM is a parallelogram.

The line through L parallel to MK meets HK produced at Q.

(i) Prove that HK = KQ. **2**

(ii) Given that P is the mid-point of LQ, prove that $\angle PKQ = \angle LHQ$. **3**

(c) On a table there are two bags A and B. Bag A contains three red and five blue marbles, whilst bag B contains one red and two blue marbles.

First Ann draws one marble from bag A at random and puts it in bag B.

Then Ann draws two marbles from bag B at random without replacement.

(i) What is the probability that a marble drawn from bag A is blue? **1**

(ii) Using a tree diagram, or otherwise, find the probability that both marbles drawn from bag B are blue? **3**

Marks

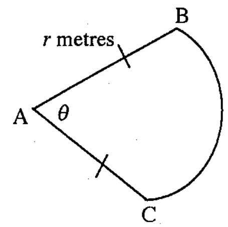
Question 9 (12 marks) **[START A NEW PAGE]**

(a) A horticulturist found the probability that a planted tomato seedling will eventually bear fruit was 0.75. He planted 'n' seedlings.

(i) What is the probability that no seedlings will bear fruit? **1**

(ii) How many seedlings must be planted to be at least 99% certain that at least one seedling will bear fruit? **3**

(b)



In the figure, AB and AC are radii of length r metres, of a circle with centre A.

The arc BC of the circle subtends an angle of θ radians at the centre A.

(i) Write down the formulae for:

(α) the length of the arc BC **1**

(β) the area of the sector ABC **1**

(ii) The perimeter of the sector ABC is 12 metres.

(α) Show that the area Y of the sector is given by $Y = \dfrac{72\theta}{(\theta+2)^2}$ **2**

(β) Hence, show that the maximum area of the sector is 9 square metres. **4**

Question 10 (12 marks) **[START A NEW PAGE]** **Marks**

(a) (i) A mathematically-minded ant of negligible size sets out on a walking adventure on a number plane. **4**

Starting at the origin, it walks out a distance of one unit in the positive direction along the x-axis.

It then turns left 90° and goes up half a unit from its current point.

If the ant continues turning left 90°, going half of the distance it previously went and keeps repeating this pattern, determine the coordinates where the ant will eventually end up.

(ii) Explain why these coordinates can only be considered as close approximations. **1**

(b) Robert takes out a \$250 000 mortgage and agrees to pay the bank a \$2000 installment each month. The interest rate on the loan is 7.2% per annum, compounded monthly, and the contact requires that the loan is paid off within 20 years.

It can be shown that the amount A_n owing after the n[th] repayment is given by the formula:

$$A_n = P(1+r)^n - \frac{M\left((1+r)^n - 1\right)}{r}$$

where P is amount borrowed

r is the interest rate per compounding period

M is the amount of each installment.

n is the number of compounding periods

(i) Find the amount owing on the loan at the end of the 10th year. **1**

(ii) Find A_{240} and explain why this shows that the loan is actually paid off in less than 20 years. **2**

(iii) Find how many months early that the loan is paid off. (ie. how many months below 20 years) **4**

End of Paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\ x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right)\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x>0$

2007 TRIAL HSC 2U MATHEMATICS SOLN'S

QUESTION 1

(a) $= 1.204395022$

≈ 1.20 (2dp)   * Accuracy Question

(b) $s = ut + \frac{1}{2}at^2$

$s - ut = \frac{1}{2}at^2$

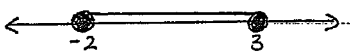
$2(s - ut) = at^2$

$\frac{2(s-ut)}{t^2} = a$

(c) $-5 \le 2m - 1 \le 5$

$-4 \le 2m \le 6$

$-2 \le m \le 3$



(d) $G(x) = 1 - x^2$

$G(3) = 1 - 3^2 = -8$

$G(0) = 1 - 0^2 = 1$

$G(3) - G(0) = -8 - 1$

$= -9$

(e) $= 3(x^2 - 4y^2)$

$= 3(x - 2y)(x + 2y)$

(f) $= P(R_A N_B) + P(N_A R_B)$

$= \frac{2}{10} \times \frac{1}{4} + \frac{3}{10} \times \frac{3}{4}$

$= \frac{2}{5}$

QUESTION 2

$y = x^2 - 3x + 2$

$\frac{dy}{dx} = 2x - 3$

at $x = 3$ $\frac{dy}{dx} = 2(3) - 3$

$= 3 = m$

then $m = 3$ pt $(3, 2)$

$y - 2 = 3(x - 3)$

$y - 2 = 3x - 9$

$y = 3x - 7$ or $3x - y - 7 = 0$

(b)(i) $y = \tan(\ln x)$

$\frac{dy}{dx} = \frac{1}{x}\sec^2(\ln x)$

(ii) $y = x(1+x)^{\frac{1}{2}}$

$\frac{dy}{dx} = (1+x)^{\frac{1}{2}} \cdot 1 + x \cdot \frac{1}{2}(1+x)^{-\frac{1}{2}}$

$= \sqrt{1+x} + \frac{x}{2\sqrt{1+x}}$

(c)(i) $\int \frac{2x}{x^2+1}\,dx = \ln(x^2+1) + c$

(ii) $\int_{\frac{\pi}{3}}^{\pi} \cos\left(\frac{1}{2}x\right)dx$

$= 2\left[\sin\left(\frac{1}{2}x\right)\right]_{\frac{\pi}{3}}^{\pi}$

$= 2\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right]$

$= 2\left[1 - \frac{1}{2}\right]$

$= 1$

(d) $f(x) = x^2 + x$

$f'(x) = 2x$

$f''(x) = 2$

$f(b) = b^2 + b$

$f''(b) = 2$

$2 = b^2 + b$

$0 = b^2 + b - 2$

$0 = (b + 2)(b - 1)$

$\therefore b = -2, 1$

QUESTION 3

(a) $\log_2 48 - \log_2 4 - \log_2 3$

$= \log_2 48 - (\log_2 4 + \log_2 3)$

$= \log_2\left(\frac{48}{4 \times 3}\right)$

$= \log_2 4$

$= 2\log_2 2$

$= 2$

(b) $2y = -3x + 12$ $ay = -2x + b$

$y = -\frac{3}{2}x + 6$ $y = -\frac{2}{a}x + \frac{b}{a}$

$m_1 = -\frac{3}{2}$ $m_2 = -\frac{2}{a}$

$\perp \Rightarrow m_1 \times m_2 = -1$

$-\frac{3}{2} \times -\frac{2}{a} = -1$

$\frac{6}{2a} = -1$

$6 = -2a$

$a = -3$

sub $(2,3)$ $2(2) + -3(3) = b$

$4 - 9 = b$

$b = -5$

$a = -3$, $b = -5$

(c) $a + a + d + a + 2d = 27$

$3a + 3d = 27$

$a + d = 9 \Rightarrow a = 9 - d$

$a + 3d + a + 4d + a + 5d = 63$

$3a + 12d = 63$

$a + 4d = 21$

$(9 - d) + 4d = 21$

$9 + 3d = 21$

$3d = 12$

$d = 4$ $a = 9 - 4$

$a = 5$

(d) $\sum_{r=1}^{20} (2r+1) = 3 + 5 + 7 + \dots 41$

$S_{20} = \frac{20}{2}(3 + 41)$

$= 10 \times 44$

$= 440$

(e) $0.\dot{3}\dot{0} = 0.30 + 0.0030 + 0.000030 \dots$

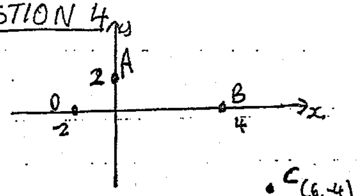
$a = 0.30$ $r = 0.01$ $n = \infty$

$S_\infty = \frac{0.30}{1 - 0.01}$

$= \frac{30}{99}$

$= \frac{10}{33}$

QUESTION 4



(i) $d_{AB} = \sqrt{(4-0)^2 + (0-2)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

(ii) $m_{AB} = \frac{0-2}{4-0}$

$= -\frac{1}{2}$

(iii) $l \Rightarrow m = -\frac{1}{2}$ $C(6, -4)$

$y - -4 = -\frac{1}{2}(x - 6)$

$2y + 8 = -x + 6$

$x + 2y + 2 = 0$

(iv) $D \Rightarrow x$-int $\Rightarrow y = 0$

$x + 2(0) + 2 = 0$

$x = -2$ $\therefore (-2, 0)$

(v) $d = \frac{|1(0) + 2(2) + 2|}{\sqrt{1^2 + 2^2}}$

$= \frac{|6|}{\sqrt{5}}$

$= \frac{6}{\sqrt{5}}$

(vi) ABCD $\Rightarrow$ Trapezium

$d_{DC} = \sqrt{(6 - -2)^2 + (-4 - 0)^2}$

$= \sqrt{80}$

$= 4\sqrt{5}$

$A = \frac{1}{2} \times \frac{6}{\sqrt{5}} \times \left[2\sqrt{5} + 4\sqrt{5}\right]$

$= \frac{3}{\sqrt{5}} \times \frac{6\sqrt{5}}{1}$

$= 18\,u^2$

QUESTION 5

(a) LHS $= \sin^2 A + 2\sin A\cos A + \cos^2 A$

$+ \sin^2 A - 2\sin A\cos A + \cos^2 A$

$= 2\sin^2 A + 2\cos^2 A$

$= 2(\sin^2 A + \cos^2 A)$

$= 2 \times 1 = 2 =$ RHS

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

c) $A = \frac{1}{2} \times 4 \times 8 \times \sin 30°$

$= 8 cm^2$

d) $36° = \frac{36\pi}{180} = \frac{\pi}{5}$

$A = \frac{1}{2} \times 10^2 \left(\frac{\pi}{5} - \sin\frac{\pi}{5}\right)$

$= 10\pi - 50\sin\frac{\pi}{5}$

$= 2.026663921$

$= 2 cm^2$ (n.cm)

(e)(i) $y = \sin x$ $y = \cos x$

$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

(ii) $A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx$

$= \left[-\cos x - \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$

$= -\left[\left[\cos x + \sin x\right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}\right]$

$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$

$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$

$= -\left[-\frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}}\right]$

$= \frac{4}{\sqrt{2}} u^2$

QUESTION 6

(a) Real roots ⇒ $\Delta \geq 0$

$a = k$ $b = -8$ $c = k$

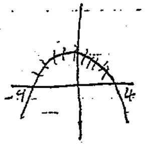
$\Delta = (-8)^2 - 4(k)(k)$

$= 64 - 4k^2$

$64 - 4k^2 \geq 0$

$4(16 - k^2) \geq 0$

$4(4 - k)(4 + k) \geq 0$



(b)(i) $f(x) = 2x^3 + ax^2 + bx + 3$

$f'(x) = 6x^2 + 2ax + b$

$f'(-2) = 6(-2)^2 + 2(a)(-2) + b$

$= 24 - 4a + b$

$f'(1) = 6(1)^2 + 2(a)(1) + b$

$= 6 + 2a + b$

$24 - 4a + b = 0$ $6 + 2a + b = 0$

$-4a + b = -24$ $2a + b = -6$

$-4a + b = -24$
$2a + b = -6$ } −

$-6a = -18$

$a = 3$

$2(3) + b = -6$

$6 + b = -6$

$b = -12.$

(ii) $f(1) = 2(1)^3 + 3(1)^2 - 12(1) + 3$

$= -4$ $(1, -4)$

$f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 3$

$= 23$ $(-2, 23)$

$f''(x) = 12x + 6$

$f''(1) = 12(1) + 6$

$= 18 > 0$ ∪ ∴ Min Pt $(1, -4)$

$f''(-2) = 12(-2) + 6$

$= -18 < 0$ ∩ ∴ Max Pt $(-2, 23)$

(iii) $f''(x) = 12x + 6 = 0$

$12x + 6 = 0$

$12x = -6$

$x = -\frac{1}{2}$

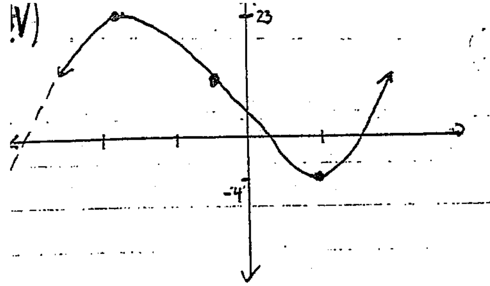
$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)^2 - 12($[illegible]

$= 9\frac{1}{2}$ $\left(-\frac{1}{2}, 9\right.$[illegible]

x	-1	$-\frac{1}{2}$	0
$f''(x)$	$-12+6$	$-6+6$	$0+6$
	$= -6 < 0$	$= 0$	$= 6 > 0$

Change of concavity ∴ Pt of Inflexion

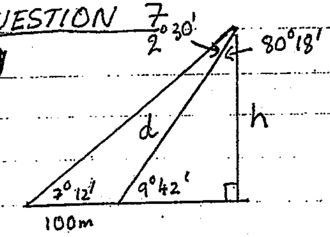
(iv)



(v) $x > -\frac{1}{2}$

QUESTION 7

(a)(i)



$\frac{d}{\sin 7°12'} = \frac{100}{\sin 2°30'}$

$d = \frac{100 \sin 7°12'}{\sin 2°30'}$

$\sin 9°42' = \frac{h}{d}$

$h = d \times \sin 9°42'$

$h = \frac{100 \sin 7°12' \cdot \sin 9°42'}{\sin 2°30'}$

(ii) $h = 48.41268995$

$= 48.4$ m (3sf)

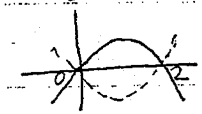
(b)(i) x-Int ⇒ $y = 0$

$0 = 2x - x^2$

$0 = x(2 - x)$

$x = 0$ $x = 2$

(ii)



$V = \pi \int_0^2 (2x - x^2)^2\,dx$

$= \pi \int_0^2 (4x^2 - 4x^3 + x^4)\,dx$

$= \pi \left[\frac{4x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}\right]_0^2$

$= \pi \left[\left(\frac{32}{3} - 16 + \frac{32}{5}\right) - (0)\right]$

$= \pi \left(\frac{16}{15}\right)$

$= \frac{16\pi}{15} u^3$

(c)(i)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0

(ii) $A \doteqdot \frac{\frac{\pi}{4}}{3}\left[0 + 0 + 4\left(\frac{1}{2} + \frac{1}{2}\right) + 2(1)\right]$

$\doteqdot \frac{\pi}{12} \times 6$

$\doteqdot \frac{\pi}{2}$

QUESTION 8

(a)(i) Stat Pts at $x = 2$ & $x = 6$

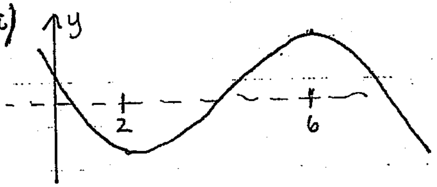
x	1	2	3
$f'(x)$	−	0	+
	\	—	/

Min at $x = 2$

x	5	6	7
$f'(x)$	+	0	−
	/	—	\

Max at $x =$[illegible]

(ii)



(b)(i) ML ∥ KQ (given)

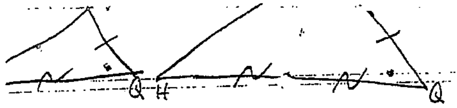
MK ∥ LQ (given)

KQLM is ∥gram (Opp sides ∥)

ML = KQ (opp sides equal KQLM)

ML = HK (opp sides equal HKLM)

∴ HK = KQ



$PQK = \angle LQH$ (common $\angle$)

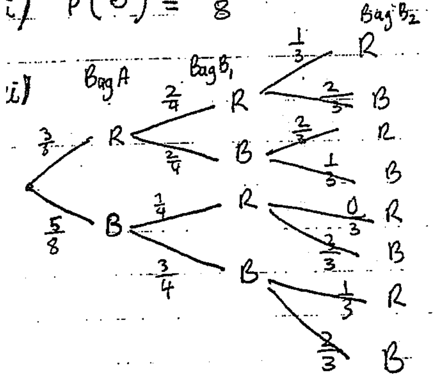
$\frac{QK}{PH} = \frac{QP}{QL} = \frac{1}{2}$ ($HK = KQ$, $LP = PQ$)

$\Delta QPK \,|||\, \Delta QLH$ (side in ratio & included $\angle$ equal)

$\angle PKQ = \angle LHQ$ (Corr $\angle$s in $\Delta QPK \,|||\, \Delta QLH$)

i) $P(B) = \frac{5}{8}$

ii)



) (both Blue from Bag B)

$P(RBB) + P(BBB)$

$\frac{3}{8} \times \frac{2}{4} \times \frac{1}{3} + \frac{5}{8} \times \frac{3}{4} \times \frac{2}{3}$

$\frac{3}{8}$

QUESTION 9

(i) $P(\text{no fruit}) = 1 - 0.75$
$= 0.25$

n seedlings no fruit
$= (0.25)^n$

ii) 99% certain
$P(\text{at least one fruit}) = 0.99$

At least one $= 1 -$ none

$(0.25)^n \leq 1 - 0.99$

~~$n \ln 0.25 \leq \ln 0.01$~~

$n \geq \frac{\ln 0.01}{\ln 0.25}$

$n \geq 3.32$

$\therefore n = 4$

(b)(i)(α) $BC = r\theta$

(β) $A = \frac{1}{2} r^2 \theta$

(ii)(α)
Given $P = 2r + r\theta = 12$

$r(2+\theta) = 12$

$r = \frac{12}{(2+\theta)}$

$Y = \frac{1}{2}\left(\frac{12}{2+\theta}\right)^2 \theta$

$= \frac{1}{2}\left(\frac{144}{(2+\theta)^2}\right)\theta$

$= \frac{72\theta}{(2+\theta)^2}$

(β) Max $\Rightarrow \frac{dY}{d\theta} = 0$

$Y = \frac{72\theta}{(2+\theta)^2}$

$\frac{dY}{d\theta} = \frac{(2+\theta)^2 \cdot 72 - 72\theta \cdot 2(2+\theta)}{((2+\theta)^2)^2}$

$= \frac{72(2+\theta)^2 - 144\theta(2+\theta)}{(2+\theta)^4}$

$= \frac{72(2+\theta)[(2+\theta) - 2\theta]}{(2+\theta)^4}$

$= \frac{72(2-\theta)}{(2+\theta)^3}$

$\therefore 0 = \frac{72(2-\theta)}{(2+\theta)^3}$ $\therefore \theta = 2$

when $\theta = 2$

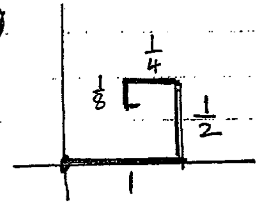
θ	1	2	3
$\frac{dY}{d\theta}$	$\frac{72(2-1)}{(2+1)^3}$	0	$\frac{72(2-3)}{(2+3)^3}$
	$\frac{72}{3} > 0$	0	$\frac{-72}{125} < 0$
	/	—	\

$\therefore$ Max value at $\theta = 2$

At $x = 2$ $Y = \frac{72(2)}{(2+2)^2}$
$= \frac{144}{16}$
$= 9\text{ m}^2$

QUESTION 10

(a)(i)



$\overleftrightarrow{GP_1}$ $1, -\frac{1}{4}, \frac{1}{16}, \ldots$ $a = 1$, $r = -\frac{1}{4}$

$GP_2 \updownarrow$ $\frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, \ldots$ $a = \frac{1}{2}$, $r_2 = -\frac{1}{4}$

$S_{1\infty} = \frac{1}{1 - -\frac{1}{4}} = \frac{4}{5}$

$S_{2\infty} = \frac{\frac{1}{2}}{1 - -\frac{1}{4}} = \frac{2}{5}$

$\therefore \left(\frac{4}{5}, \frac{2}{5}\right)$

(ii) Ant will only approach point $\left(\frac{4}{5}, \frac{2}{5}\right)$ as both r_1 & r_2 $(-1 \leq r \leq 1)$ imply a limiting sum.

12 = 240

$M = 2000$

(i) $A_{120} = 250000(1.006)^{120} - \frac{2000(1.006^{120} - 1)}{0.006}$

$= 162\,498.4953$

$= \$162\,498.50$ (2dp)

(ii) $A_{240} = 250000(1.006)^{240} - \frac{2000(1.006^{240} - 1)}{0.006}$

$= -16\,881.16943$

Since A_{240} is negative, then Robert has paid more than he owed, ie loan is in credit after 20yrs. The loan was paid off in less than 20 years.

(iii) $A_n = 0$

$0 = 250000(1.006)^n - \frac{2000(1.006^n - 1)}{0.006}$

$= 250000(1.006)^n - 333333.\dot{3}(1.006)^n + 333333.\dot{3}$

$= 333333.\dot{3} - 83333.\dot{3}(1.006)^n$

$83333.\dot{3}(1.006)^n = 333333.\dot{3}$

$(1.006)^n = \frac{333333.\dot{3}}{83333.\dot{3}}$

$1.006^n = 4$

$n \ln 1.006 = \ln 4$

$n = \frac{\ln 4}{\ln 1.006}$

$= 231.7415\ldots$

$= 232$ months

$\therefore 240 - 232 = 8$ months

Loan was paid off 8 months early.